

P425/1
PURE MATHEMATICS
Paper 1
Nov./ Dec. 2017
3 hours



UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer all the eight questions in section A and any five from section B.

Any additional question(s) answered will not be marked.

All necessary working must be shown clearly.

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent non programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40 MARKS)

Answer all questions in this section.

1. The coefficients of the first three terms of the expansion of $\left(1 + \frac{x}{2}\right)^n$ are in an Arithmetic Progression (AP). Find the value of n . (05 marks)
2. Solve the equation $3\tan^2\theta + 2\sec^2\theta = 2(5-3\tan\theta)$ for $0^\circ \leq \theta \leq 180^\circ$. (05 marks)
3. Differentiate $\left(\frac{1+2x}{1+x}\right)^2$ with respect to x . (05 marks)
4. Solve for x in the equation $4^{2x} - 4^{x+1} + 4 = 0$. (05 marks)
5. The vertices of a triangle are $P(4,3)$, $Q(6,4)$ and $R(5,8)$. Find angle RPQ using vectors. (05 marks)
6. Show that $\int_2^4 x \ln x \, dx = 14\ln 2 - 3$. (05 marks)
7. The equation of a curve is given by $y^2 - 6y + 20x + 49 = 0$.
 - (a) Show that the curve is a parabola. (03 marks)
 - (b) Find the coordinates of the vertex. (02 marks)
8. A container is in the form of an inverted right circular cone. Its height is 100 cm and base radius is 40 cm. The container is full of water and has a small hole at its vertex. Water is flowing through the hole at a rate of $10 \text{ cm}^3 \text{ s}^{-1}$. Find the rate at which the water level in the container is falling when the height of water in the container is halved. (05 marks)

SECTION B: (60 MARKS)

Answer any five questions from this section. All questions carry equal marks.

9. (a) Given that the complex number Z and its conjugate \bar{Z} satisfy the equation $Z\bar{Z} - 2Z + 2\bar{Z} = 5 - 4i$, find the possible values of Z . (06 marks)
- (b) Prove that if $\frac{Z-6i}{Z+8}$ is real, then the locus of the point representing the complex number Z is a straight line. (06 marks)
10. A circle whose centre is in the first quadrant touches the x - and y -axes and the line $8x - 15y = 120$. Find the:
- (a) equation of the circle. (10 marks)
- (b) point at which the circle touches the x -axis. (02 marks)
11. A curve whose equation is $x^2y + y^2 - 3x = 3$ passes through points $A(1,2)$ and $B(-1,0)$. The tangent at A and the normal to the curve at B intersect at point C . Determine the:
- (a) equation of the tangent. (06 marks)
- (b) coordinates of C . (06 marks)
12. (a) Express $\cos(\theta + 30^\circ) - \cos(\theta + 48^\circ)$ in the form $R\sin P\sin Q$, where R is a constant. Hence solve the equation.
 $\cos(\theta + 30^\circ) - \cos(\theta + 48^\circ) = 0.2$ (06 marks)
- (b) Prove that in any triangle ABC , $\frac{\sin(A-B)}{\sin(A+B)} = \frac{a^2 - b^2}{c^2}$. (06 marks)
13. (a) Solve for x and y in the following simultaneous equations.
 $(x - 4y)^2 = 1$
 $3x + 8y = 11$ (06 marks)
- (b) Find the set of values of x for which $4x^2 + 2x < -3x + 6$. (06 marks)

14. (a) The points A and B have position vectors a and b . A point C with position vector c lies on AB such that $\frac{AC}{AB} = \lambda$.
Show that $c = (1 - \lambda)a + \lambda b$. (04 marks)

- (b) The vector equations of two lines are
 $r_1 = 2i + j + \lambda(i + j + 2k)$ and
 $r_2 = 2i + 2j + t k + \mu(i + 2j + k)$

where i, j and k are unit vectors and λ, μ and t are constants.
Given that the two lines intersect, find

- (i) the value of t .
(ii) the coordinates of the point of intersection. (08 marks)

15. (a) Sketch the curve $y = x^3 - 8$. (08 marks)

- (b) The area enclosed by the curve in (a), the y -axis and the x -axis is rotated about the line $y = 0$ through 360° . Determine the volume of the solid generated. (04 marks)

16. (a) Solve the differential equation

$$\frac{dy}{dx} = (xy)^{1/2} \ln x, \text{ given that } y = 1 \text{ when } x = 1.$$

Hence find the value of y when $x = 4$. (12 marks)